

Plasmon exchange model for superconductivity in Carbon nanotubes

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Abstract

Recent investigations of superconductivity in carbon nanotubes have shown that a single-wall zigzag nanotube can become superconducting at around 15 K. Theoretical studies of superconductivity in nanotubes using the traditional phonon exchange model, however, give a superconducting transition temperature T_c less than 1K. To explain the observed higher critical temperature we explore the possibility of the plasmon exchange mechanism for superconductivity in nanotubes. We first calculate the effective interaction between electrons in a nanotube mediated by plasmon exchange and show that this interaction can become attractive. Using this attractive interaction in the modified Eliashberg theory for strong coupling superconductors, we then calculate the critical temperature T_c in a nanotube. We find that T_c is sensitively dependent on the dielectric constant of the medium, the effective mass of the electrons and the radius of the nanotube. Our theoretical results can explain the observed T_c in a nanotube.

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1 Introduction

Carbon nanotubes, first discovered by Iijima in 1991 [1], are a new form of carbon with exotic physical properties [2]. Depending upon their helicity and chirality the electronic and transport properties of the carbon nanotubes vary in spectacular ways [3, 4, 5, 6, 7]. It has been shown both theoretically and experimentally that nanotubes of zero helicity are predominantly metallic in character whereas nanotubes of nonzero helicity are mostly semiconducting in character [4, 8, 9]. Using different methods of synthesis good quality single-wall, multi-wall as well as bundles of nanotubes have now been produced [10, 11]. The diameter of a carbon nanotube is of the order of nanometer and they are upto several microns in length. The question whether a carbon nanotube or a group of carbon nanotubes can exhibit superconductivity has been addressed in several recent studies. Carbon nanotubes are observed to pass supercurrent between superconducting leads due to proximity effect [12]. Recent experiments by Tang *et al.* [13] have shown the presence of superconductivity in single-wall zigzag nanotubes of radius 2.1 \AA at about 15 K. On the other hand Kochiak *et al.* [14] have reported superconductivity in a bundle of arm-chair nanotubes of radius 7 \AA at about 0.55 K. There have been some theoretical explanations of superconductivity in carbon nanotubes and the origin of superconducting fluctuations [15, 16]. Sedeki *et al.* [17] used momentum space renormalization group theory to study the influence of phonons and the Coulomb interaction on the superconducting response function of armchair single-wall nanotubes. They found that the superconducting fluctuations due to phonons can be easily destroyed by Coulomb repulsion. Gonzalez [18] has recently pointed out that an electron-phonon mechanism of superconductivity in ropes of carbon nanotubes can give a superconducting transition temperature T_c less than 1K. It appears that the phonon exchange mechanism can not account for superconductivity in a single-wall carbon nanotube (SWNT) observed at finite temperature. In this paper we introduce the plasmon exchange mechanism for superconductivity in a metallic carbon nanotube with the expectation that a plasmon with its frequency higher than the phonon frequency would give a higher critical temperature in a carbon nanotube. We first calculate the effective interaction between electrons in a nanotube mediated by plasmon exchange and show that this interaction can be attractive. We then use this effective interaction in the Eliashberg theory [19] of strong coupling superconductors

as modified by McMillan [20] to calculate the superconducting transition temperature, T_c . In section 2 we introduce the plasmon exchange model for superconductivity in a metallic carbon nanotube and show the details of our calculations. This model was previously used by Longe and Bose [21] to calculate the critical temperature in high- T_c superconductors. In section 3 we present our results and discussions. Finally in section 4 we present our conclusions.

2 The model

Since we are going to present the plasmon-exchange model of superconductivity in a SWNT we first review briefly the excitation of a plasmon in a metallic nanotube. In our model we consider that the length of a carbon nanotube is very large (several microns) compared to its radius a (several angstroms). We assume that the electrons can move parallel to the axis of the tube described by the quantum number q as well as around the tube axis described by the azimuthal quantum number μ . The dielectric function $\epsilon(Q, \omega)$ of the nanotube is calculated in the random phase approximation (RPA) [22] using

$$\epsilon(Q, \omega) = \epsilon + v_o(Q)\Pi(Q, \omega), \quad (1)$$

where $Q = [q, \mu/a]$, q and μ/a are the components of the wave vector for motions parallel and azimuthal directions, respectively. The polarization propagator in the frequency region of plasmon excitation has been shown to be

$$\Pi(Q, \omega) \approx -\frac{n_s Q^2}{m\omega^2}, \quad (2)$$

where n_s and m are surface number density and effective mass of the electron, respectively. In Eq. (1) ϵ is the dielectric constant of the medium and $v_o(Q)$ is the bare Coulomb interaction between two electrons on a nanotube and is given by

$$v_o(Q) = 4\pi e^2 a I_\mu(aq) K_\mu(aq) \quad (3)$$

where $I_\mu(aq)$ and $K_\mu(aq)$ are modified Bessel functions, e is the electronic charge and the azimuthal quantum number μ runs through all integral values. The plasmon frequencies are obtained from the zeros of the dielectric function as

$$[\omega_\mu(q)]^2 = \frac{4\pi n_s e^2 a}{m\epsilon} Q^2 I_\mu(aq) K_\mu(aq) \quad (4)$$

In reference 22 it has been shown that the plasmon frequency corresponding to $\mu = 0$ is semi-acoustic in nature whereas the frequencies for $\mu \neq 0$ are optical. Once the dielectric function of the nanotube $\epsilon(Q, \omega)$ has been determined by Eq. (1), we can write the effective interaction between two electrons on a nanotube due to plasmon exchange as

$$V(Q, \omega) = \frac{v_o(Q)}{\epsilon(Q, \omega)} = \frac{v_o(Q)}{\epsilon + v_o(Q)\Pi(Q, \omega)} \quad (5)$$

Substituting for $\Pi(Q, \omega)$ from Eq.(2) we can rewrite Eq. (5) as

$$V(Q, \omega) = \frac{v_o(Q)}{\epsilon} + \frac{v_o^2(Q)n_s Q^2}{\epsilon^2 m \omega^2 - v_o(Q)\epsilon n_s Q^2} \quad (6)$$

The first term on the right hand side of Eq. (6) is the statically screened Coulomb repulsion part and the second term represents the effect of plasmon excitation. We notice that the second term can become attractive and can thus lead to superconductivity in a nanotube.

To examine how this effective interaction can lead to superconductivity in a nanotube, we use the Eliashberg model [19] of superconductivity in a strong-coupling superconductor. Although in its original form the Eliashberg model is a numerical model, many analytic approximations have been presented by McMillan and others [20, 23]. In this paper we use the McMillan model which gives the critical temperature for superconductivity in a strong coupling superconductor as

$$T_c = \frac{\langle \omega \rangle}{1.45} \exp\left[-\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)}\right] \quad (7)$$

In this equation $\langle \omega \rangle$ is the average value of the frequency of the boson, the exchange of which is responsible for superconductivity, λ is the coupling strength due to attractive part of the effective interaction and μ^* is the Coulomb repulsion parameter. It has been shown by Allen and Dynes [24] that if the effective interaction between electrons in a superconductor can be written as

$$V(Q, \omega) = v_o(Q) + \frac{2\omega(Q)|M(Q)|^2}{\omega^2 - \omega^2(Q)}, \quad (8)$$

then the above parameters can be obtained from

$$\lambda = \lambda(0) = N(0) < 2 \frac{|M(Q)|^2}{\omega(Q)} >_{FS} \quad (9)$$

and

$$\lambda < \omega^2 > = N(0) < 2|M(Q)|^2 \omega(Q) >_{FS} \quad (10)$$

where $N(0)$ is the density of states of the electrons at the Fermi surface and $\langle \dots \rangle_{FS}$ indicates that an average of the expression is taken over the Fermi surface. Combining Eqs. (9) and (10) one can obtain $\langle \omega \rangle$ from

$$\langle \omega \rangle = \sqrt{\frac{\lambda \langle \omega^2 \rangle}{\lambda}} \quad (11)$$

We can express our effective interaction (Eq. (6)) in the form of Eq. (8) if we identify

$$\omega^2(Q) = \frac{n_s Q^2 v_o(Q)}{m\epsilon} \quad (12)$$

and

$$|M(Q)|^2 = \frac{1}{2} \sqrt{\frac{n_s Q^2 v_o^3(Q)}{m\epsilon^3}} \quad (13)$$

Also for the carbon nanotubes where the electrons have axial and azimuthal motions, the Fermi surface will be cylindrical and the density of states at the Fermi surface will be given by

$$N(0) = \frac{m}{2\pi^2 a} \sum_{\mu} \frac{1}{\sqrt{k_F^2 - (\frac{\mu}{a})^2}} \quad (14)$$

Substituting the values of $\omega^2(Q)$, $|M(Q)|^2$ and $N(0)$ in Eqs. (9) and (10) and carrying out the average over the Fermi surface we have calculated the value of $\lambda \langle \omega^2 \rangle$ and λ and then $\langle \omega \rangle$ from Eq. (12). These parameters obviously depend on the dielectric constant ϵ , the effective mass m , the surface number density n_s of the electron and the radius a of the nanotube. The Coulomb repulsion parameter μ^* depends on other boson frequencies and like many other investigators [20, 25] we take its numerical value to be 0.1. Substituting these values of $\langle \omega \rangle$, λ and μ^* in the McMillan's expression [Eq. (7)] for T_c , we obtain the critical temperature as a function of the parameters ϵ , m , a and n_s .

3 Results and discussions

To calculate the critical temperature T_c in a nanotube one needs to know the values of the parameters ϵ , a , $Z = m/m_e$ (m_e being the mass of the bare electron) and n_s . It turns out that in a metallic nanotube the number density of electrons n_s is fixed and is independent of whether it is an arm-chair or zigzag nanotube. Assuming that each carbon atom in such a nanotube contributes one electron to the conduction band, n_s can be shown to be

$3.73 \times 10^{15}/\text{cm}^2$. The values of the other parameters are not fixed and are known only approximately. The radii a of a nanotube is known to vary from 0.2 to 1.0 nm. The effective dielectric constant ϵ has been reported to be of the order of 1.4 [26]. The effective mass Z has been reported to be of the order of that in a graphite sheet which is known to be 0.24 in a zigzag nanotube and speculated to be one order larger in an arm-chair nanotube. Since these parameters are not known exactly, we thought it would be interesting to study numerically how T_c varies as a function of their reasonable (measured or speculated) values. To get a better understanding of ϵ and Z dependence of T_c , in Figure 1 we present a contour plot of T_c as a function of ϵ and Z for $a = 0.21\text{nm}$ corresponding to a zigzag nanotube. The figure clearly shows that T_c decreases with increasing ϵ and decreasing Z . In Figure 2 we have plotted T_c versus ϵ for $Z = 0.24, 0.26, 0.28$ and 0.30 for a nanotube of radius $a = 0.21\text{nm}$ and in Fig. 3 we have plotted T_c versus Z for $\epsilon = 1.3, 1.35, 1.40$ and 1.45 for the same nanotube.

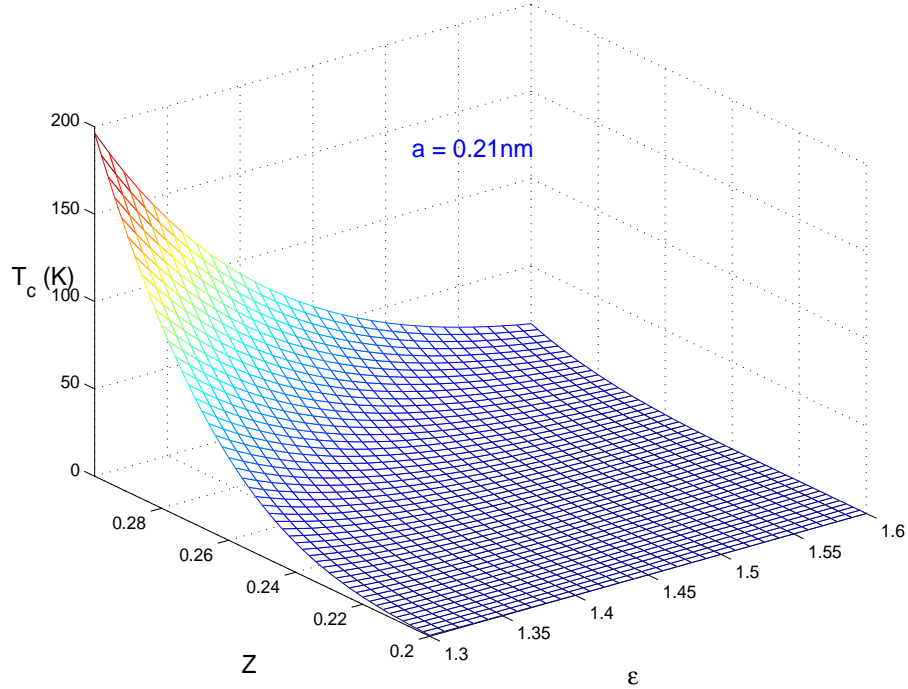


Figure 1: Variation of T_c as a function of ϵ and Z .

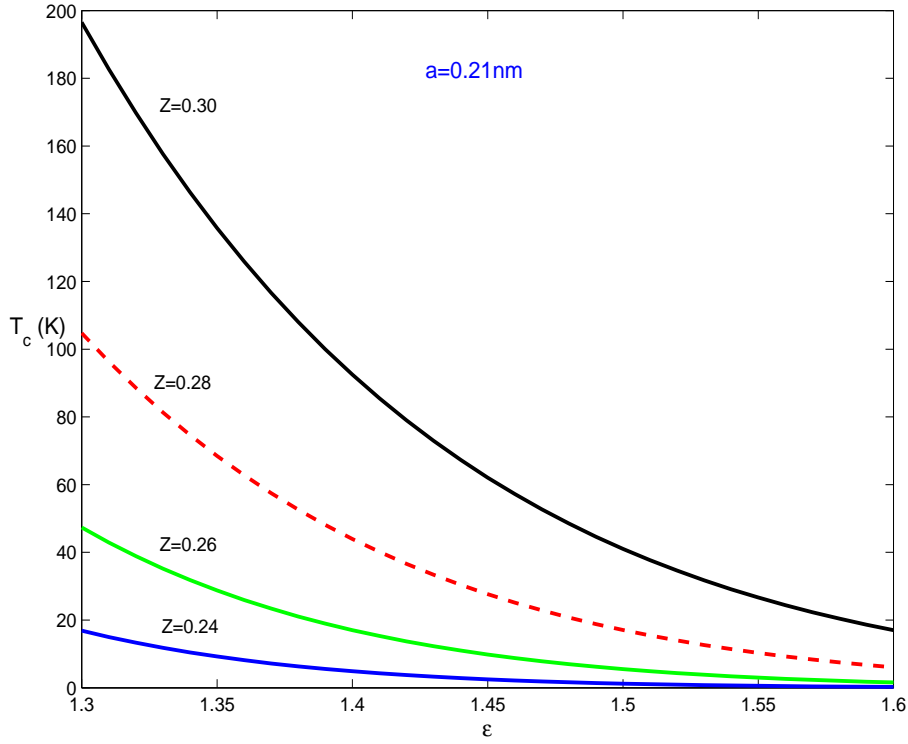


Figure 2: T_c as a function of ϵ for $Z = 0.24, 0.26, 0.28$ and 0.30 .

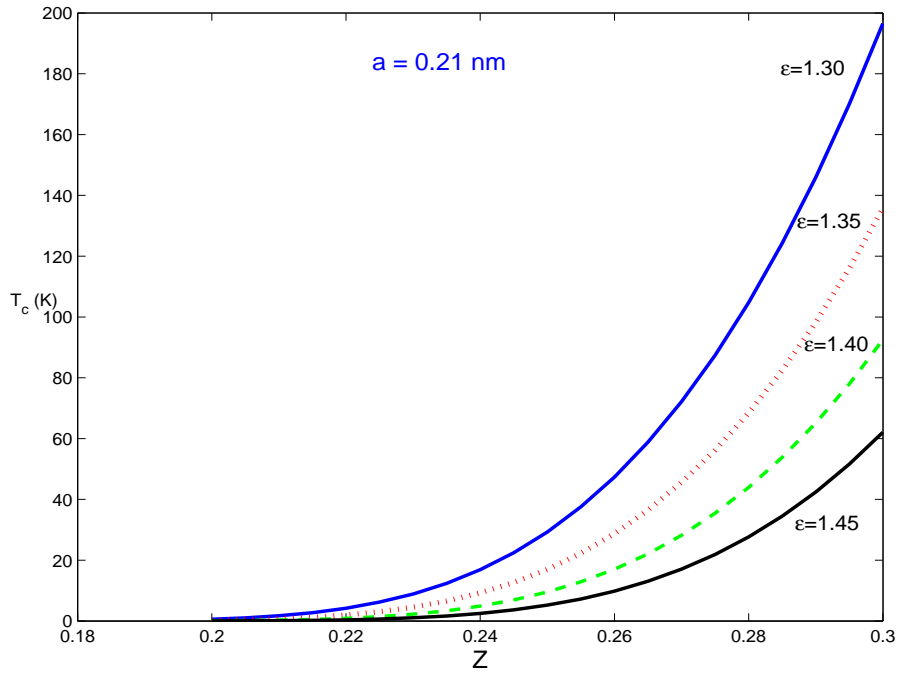


Figure 3: T_c as a function of Z for $\epsilon = 1.30, 1.35, 1.40$ and 1.45 .

These figures make it abundantly clear that the plasmon exchange model for superconductivity shows that the critical temperature in a nanotube is

indeed sensitively dependent on the parameters ϵ and Z for a fixed a and depending on their actual values T_c can lie within a wide range. For example we find that for $\epsilon = 1.3$ and $Z = 0.24$, $T_c = 17$ K which is close to what Tang *et al.* [13] have measured in a single-wall zigzag nanotube. However, for $\epsilon = 1.45$ and $Z = 0.24$, T_c can be as low as 2.5 K.

4 Summary and Conclusions

In this paper we have studied the plasmon exchange model for superconductivity in a single-wall carbon nanotube. We have first shown that the effective interaction between two electrons mediated by plasmon exchange can become attractive which in its turn can lead to superconductivity in a nanotube. The superconducting critical temperature is then calculated by using Eliashberg theory for strong coupling superconductors as modified by McMillan and others. The critical temperature is found to be sensitively dependent on ϵ , the dielectric constant of the medium; m_e , the effective mass of the electron; and a , the radius of the nanotube. For reasonable values of these parameters the calculated value of T_c is found to be in reasonable agreement with the experimental values of a zigzag nanotube [13].

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